METHOD FOR DESCRIPTION OF CREEP AND LONG-TERM STRENGTH WITH PURE ELONGATION

A. M. Lokoshchenko and S. A. Shesterikov

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At the present time, there exists a relatively small number of reliable data characterizing the creep right up to fracture under a wide range of stresses. This is connected with the fact that tests with small stresses can last tens and thousands of hours. Therefore, frequently these tests are not carried to fracture (i.e., long-term strength is not considered), or they are carried to fracture without measurement of the deformations during the creep process. The boundedness and singularity of an actual material leads to a situation in which the question of the formulation of the mechanical equation of state and the kinetic equations, making it possible to take account of the process of fracture, remains open.

The present article gives the results of creep tests with pure elongation, carried out in the Institute of Mechanics of Moscow State University, and proposes a model for the description of monoaxial creep of a material right up to fracture. Tests were made of 21 samples of one melt of stainless steel Kh18N10T. The samples were tubes with an outside diameter of 12 mm, a wall thickness of 0.5 mm, and a working length of 70-100 mm. The temperature during the tests was constant and equal to 850°C. The experiments were carried out with the action of a constant elongational load on the sample. The axial deformations were determined using strain gauges, which were attached with an adhesive to elastic elements, connected with the sample and led out of the furnace. In Figs. 1-4 the continuous lines give curves of the monoaxial creep p(t) with initial stresses σ_0 equal, respectively, to 4, 5, 6, and 8 kg/mm². The mean values of the fracture time t* and the corresponding deformation time p* for each stress σ_0 are given in Table 1. If it is assumed that the dependence of the rate $\dot{p}$ of fully established creep on the stress σ is expressed by the power function $\dot{p} = A\sigma^n$ with constants of the material A and n, and if these constants are determined by the method of least squares, we obtain the following values for them: n = 3.2; A = 10^{-5} (kg/mm²)$^{-3.2}$ h^{-1}.

Following [1], we shall describe the process of fracture with creep by the introduction of the parameter $\omega(t)$, characterizing the degree of damage to the material. As usual, it is assumed that $\omega(0) = 0$ and $\omega(t^*) = 1$. We take the relationships of monoaxial creep for a material with reinforcement in the form

$$\begin{aligned} \dot{p} &= A\left[\sigma/(1-\omega^{r_1})\right]^n(1+Cp^{-\alpha}), \quad p(0)=0, \\ \dot{\omega} &= B\left[\sigma/(1-\omega^{r_2})\right]^k, \quad \omega(0)=0. \end{aligned} \tag{1}$$

The constants of the material C and α characterize the not fully established state; A and n characterize the fully established stage of the creep process; the remaining constants describe the process of the increase in the damage in the material and a transition to fracture. The values of A and n are fully determined from the condition that the rate of creep $\dot{p}$ in the fully established stage obeys a power dependence on the stress $\dot{p} = A\sigma^n$, using the method of least squares in the double logarithmic coordinates log σ, log $\dot{p}$. After this, C and α are calculated analogously for the sections of not fully established creep, whose equations, in accordance with (1), have the form

$$p(t) = [AC(\alpha+1)\sigma^n t]^{\left[\frac{1}{(\alpha+1)}\right]}.$$

In the case of no reinforcement, we obtain C = 0. With the description of creep tests, carried out with constant loads, we introduce the hypothesis of the uniform distribution of the deformations over the length of the sample; with small deformations, we will have

$$\sigma(t) = \sigma_0(1 + p(t)). \tag{2}$$

Relationships (1), taking account of (2) for a material without reinforcement, assume the following form:

$$\begin{aligned} \dot{p} &= A\left[(1+p)\,\sigma_0/(1-\omega^{r_1})\right]^n, \quad p(0)=0, \\ \dot{\omega} &= B\left[(1+p)\,\sigma_0/(1-\omega^{r_2})\right]^k, \quad \omega(0)=0. \end{aligned} \tag{3}$$

The constants A and n, as has been said above, are determined from data on fully established creep; the remaining constants B, k, r_1, and r_2 remain free parameters for the time being. For simplicity, we take $r_1 = r_2 = r$. In the investigated range of stresses (σ_0 = 4-8 kg/mm²), the rate of fully established creep $\dot{p}$ changes by an order of magnitude, while the deformation at the moment of fracture changes only insignificantly, nonmonotonically (see Table 1). In view of this, it can be

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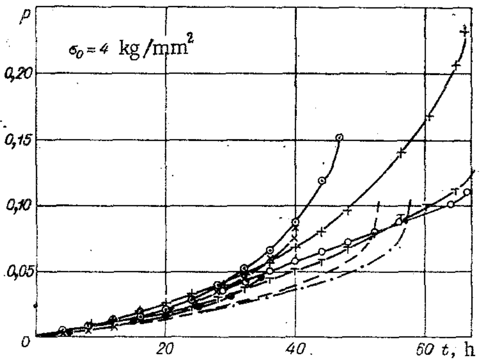

Fig. 1

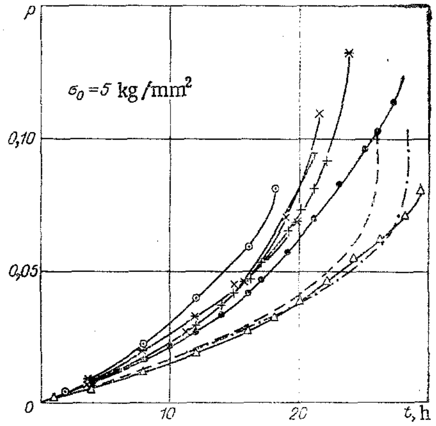

Fig. 2

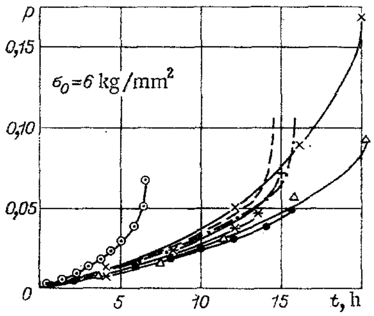

Fig. 3

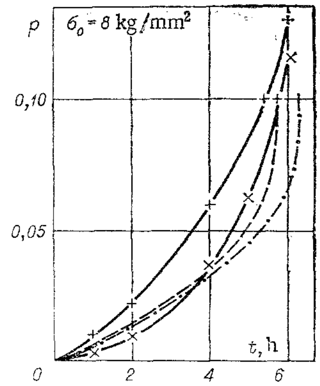

Fig. 4

assumed that $p^*(\sigma_0)$ = const. If it is postulated that k = n, this condition is satisfied. Under these circumstances, integration of the system (3) leads to a situation in which the dependences p(t) and ω(t) are produced

$$p(t) = (A/B)\omega(t),\ p^* = p(t^*) = A/B. \tag{4}$$

Averaging the values of p* over all the tests (for 21 samples), we obtain the following characteristic of the material: p* = 0.1. From (4) it follows that the coefficient B is equal to $0.95\cdot10^{-4}$ h^{-1} $(kg/mm^2)^{-3.2}$. The parameter is determined from the condition of agreement between the experimental and theoretical values of the fracture time t*. From (3) it follows

$$B\sigma_0^n t^* = I(r) = \int\limits_0^1 [(1-\omega^r)/(1+A\omega/B)]^n\, d\omega.$$

The mean value of the function I(r) is equal to 0.423. We construct the function I(r) and find that r = 2.1. The curves of the creep (the dashed lines in Fig. 1-4), corresponding to elongation of the sample by a constant force, are determined by the equation

$$t = (A\sigma_0^n)^{-1}\int\limits_0^p [1-(Bp/A)^r]^n (1+p)^{-n}\, dp. \tag{5}$$

The values of the fracture time t_1^*, calculated using (5), are given in Table 1.

In the case of elongation of the samples with constant stresses, instead of (3) we have

$$\dot{p} = A[\sigma_0/(1-\omega^r)]^n,\quad \dot{\omega} = B[\sigma_0/(1-\omega^r)]^n,\quad p(t) = (A/B)\,\omega(t). \tag{6}$$

Under these conditions, the fracture time t_2^* is determined using integration

$$t_2^* = (B\sigma_0^n)^{-1}\int\limits_0^1 (1-\omega^r)^n\, d\omega, \tag{7}$$

TABLE 1

σ_0, kg/mm²	t*, h	p*	t_1^*, h	t_2^*, h
4	54,0	0,126	53,0	58,0
5	23,5	0,100	26,0	28,0
6	15,5	0,082	14,5	16,0
8	6,0	0,124	5,8	6,3

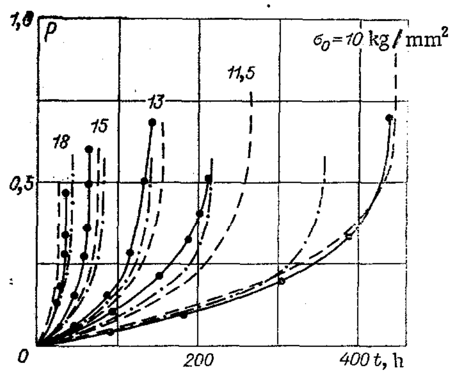

Fig. 5

TABLE 2

	σ_0, kg/mm²	10	11,5	13	15	18
g=1	t*, h	444	211	141	65	38
	p*	0,71	0,51	0,69	0,60	0,47
g=2	t*, h	445	265	155	78	26
	p*	0,90	0,78	0,69	0,60	0,50
g=3	t*, h	356	216	139	83	43
	p*	0,58	0,58	0,58	0,58	0,58

and the creep curves from the integral

$$t = (A\sigma_0^n)^{-1} \int_0^p [1 - (Bp/A)^r]^n \, dp. \tag{8}$$

The curves (8) are plotted in Figs. 1-4 by the dash–dot lines. The values of t_2^*, calculated for different stresses using (7), are given in Table 1. The change in the fracture time, due to a decrease in the area of the transverse cross section of the samples as a result of the creep of the material, is 9% on the average. It is obvious that taking account of the variability of the stresses in the case under consideration is not of fundamental importance and, from a quantitative point of view, lies within the scatter of the experimental data.

Publications [2, 3] give the results of an experimental investigation of the creep and long-term strength of titanium alloy OT-4 with constant stresses in the temperature interval 400-500°C. The continuous lines in Fig. 5 are curves of the creep corresponding to one temperature (500°C) and five different stresses σ_0 (10, 11.5, 13, 15, and 18 kg/mm²). In [4] an energy variant of the theory of creep and long-term strength is proposed, in accordance with which the curve of the creep of the material with reinforcement has the form

$$t = \frac{\sigma_0^{(m+1)}}{Ke^{\beta\sigma_0}(m+1)} \left[\left(\frac{A^*}{\sigma_0}\right)^{(m+1)} - \left(\frac{A^*}{\sigma_0} - p\right)^{(m+1)}\right], \tag{9}$$

where m, K, β, and A* are constants of the material with a given temperature. In [4] values of these quantities are given with T = 500°C; m = 3; K = 0.111 $(\text{kg/mm}^2)^4 \text{h}^{-1}$; β = 0.35 $(\text{kg/mm}^2)^{-1}$; A* = 9 kg/mm². Theoretical curves of the creep (9) with these values of the constants are plotted by the dashed lines in Fig. 5.

The experimental data [2, 3] on creep with constant stresses right up to fracture were also analyzed using the model (6). The constants entering into (6), characterizing the behavior of titanium alloy OT-4 with T = 500°C and

σ_0 = 10-18 kg/mm², were: n = 3, 59; A = 1.29·10⁻⁷ (kg/mm²)⁻³·⁵⁹h⁻¹; B = 2.21·10⁻⁷ (kg/mm²)⁻³·⁵⁹h⁻¹; r = 1.365. Theoretical curves of the creep (8), corresponding to the model (6), are plotted by the dash–dot lines in Fig. 5.

Table 2 gives values of the fracture time t* and the corresponding deformations p* with all the stresses considered. With g = 1, the above values of t* and p* were obtained in the experiments of [2, 3]. With g = 2, values of t* and p* corresponding to the last point of the curve (9) are given. With g = 3, the values of t* were calculated using (7), and the values of p* = A/B. It follows from the curves that each of the two theoretical models considered describes the experimental data of [2, 3] rather well.

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INVARIANT SOLUTIONS OF A THREE-DIMENSIONAL IDEAL PLASTICITY PROBLEM

S. I. Senashov

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1. The state of the three-dimensional flow of an incompressible plastic medium can be described by using the following system of equations [1]:

$$p_{,1} = \frac{\sqrt{2}\, k_s u_{i,jj}}{2A} - \frac{\sqrt{2}\, k_s}{A^3} e_{ij} e_{mn} u_{m,jn}, \tag{1}$$
$$u_{i,i} = 0, \quad A^2 = e_{ij} e_{ij}, \quad 2e_{ij} = u_{i,j} + u_{j,i} \ (i, j, m, n = 1, 2, 3),$$

where x_1, x_2, x_3 is a rectangular coordinate system, (u_1, u_2, u_3) is the velocity vector, p is the hydrostatic pressure, and k_s is the yield point. Summation is assumed to be over the repeated subscripts, and the subscript after the comma denotes differentiation with respect to the space variable with this subscript. Few exact solutions of this system are known at this time [2]. Axisymmetric solutions are not examined here, the papers [3-7] are devoted to them.

Let us use the method in [8] to seek particular solutions of the system (1.1). A group of continuous transformations allowed by the system (1.1) is generated by the following operators:

$$X_i = \partial/\partial x_i, \ Y_i = \partial/\partial u_i, \ M = x_i \partial/\partial x_i, \ N = u_i \partial/\partial u_i,$$
$$Z_1 = x_2 \partial/\partial x_3 - x_3 \partial/\partial x_2 + u_2 \partial/\partial u_3 - u_3 \partial/\partial u_2,$$
$$T_1 = x_2 \partial/\partial u_3 - x_3 \partial/\partial u_2, \ S = \partial/\partial p.$$

Four other operators Z_2, Z_3 and T_2, T_3 are obtained from Z_1, T_1 by circular commutation of the subscripts.

The group G_{15} is unsolvable, the operator S generates the center of this group. Let us construct optimal systems of the first, second, and third orders. They must be constructed in order to seek substantially different solutions in the group sense. Let us mention certain invariant solutions.

2. The invariant solution in the subgroup $\langle X_3 + T_1 + \alpha T_2 + \beta T_3 \rangle$ was found in [9]. It describes the flow of a prismatic rod of plastic material that is subjected to tension, torsion, and bending simultaneously.

The solution in the subgroup $\langle X_1 + T_1 + \alpha T_2,\ X_2 - T_2 + \beta T_1 \rangle$ was investigated in [10]. This solution describes the kinetic field corresponding to a homogeneous state of stress.

The invariant solution relative to the subgroup $\langle T_1 + X_1,\ \alpha X_1 + X_2 - T_2 + \beta Y_1 + \gamma Y_2 \rangle$ was studied in [11], where it is interesting in that it depends on 17 constants.

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